

Statistical and Structural Approaches to Estimate Output Gap and Inflation Forecast: The Case of Pakistan

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Abstract

The output gap and inflation forecast are important factors to analyze current state of the economy and stance of monetary policy. In this study we have measured output gap using different statistical and structural methods namely the Linear Time Trend (LTT) method, Quadratic Time Trend (QTT) method, Hodrick-Prescott (HP filter), Band Pass Baxter-King Filter (BP), Double Exponential Smoothing Method (DES), Structural Vector Autoregressive (SVAR) method. For the analysis we have used quarterly data over the period 1960 to 2014 for Pakistan. Moreover, the inflation is forecasted with univariate and multivariate models. The results suggest that Quadratic Time Trend (QTT) method and Structural Vector Autoregression (SVAR) captures the history of Pakistan economy well. Whereas, output gap estimated through Structural Vector Autoregression (SVAR) generate better inflation forecast compared to other methods.

Keywords: inflation forecast, multivariate, output gap, Pakistan

JEL Classification: C22, C53, E32

1. Introduction

The strong pressure of excess demand may provide some signal of future inflation to a central bank, which is operating in an inflation-targeting monetary policy regime. Currently, the output gap is a commonly used measure of inflationary pressure in the economy. Moreover, the output gap has a well-known role in the theoretical and applied literature in explaining price and wage inflation (De Brouwer, 1998). The output gap is defined as the difference between the actual output of the economy and its potential output. It is a temporary component of output which is usually stationary but not necessarily periodical. The potential output is the production capacity of the economy that is mostly related with permanent component of output (which is normally non-stationary) (Nelson & Plosser, 1982). The potential output of economy at any time is consistent with stable inflation (Bjørnland, Brubakk, & Jore, 2005). The output gap is positive when actual output is greater than potential output. The output gap is negative when actual output is less than potential output. A positive output gap means that there is an excess demand and rising inflation in the economy as the economy is operating above its potential level. A negative output gap means that there is an excess supply or unused capacity and falling inflation in the economy as the economy is operating below its potential level.

Since, the output gap is not directly observable, and must therefore need to be estimated by using different methods. Output gap can be measured by structural as well as statistical methods. The former includes Structural Vector Autoregression Model, while the later includes detrending method, Band pass filter, Exponential Smoothing, Hodrick-Prescott Filter. However, none of the method is free from errors. Different measures, at any particular time, may indicate different phases of economic activity. At the same time, at different times,

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different measures can be useful for inflation forecasting. Therefore, one should not rely on the results of only one method.

The specific objectives of the study are:

- To calculate the output gap with the help of both statistical and structural methods.
- To compare results of output gap with the history of Pakistan economy.
- To forecast inflation by using different measures of output gap.

The previous literature has the ability to meet just the basic requirements of business cycle for example (Arby, 2001) separate, the real GDP of Pakistan in to three component i.e. long run trend, business cycles and short run shocks by using HP filter. The study also projected GDP growth by using projected trend and business cycle. (Haider & Safdar Ullah, 2008) attempt to calculate potential output and output gap by using six different statistical and structural methods. The study found that although the output gaps calculated by using different method are not close to each other's, but there is some degree of association between them. On the basis of this association the study calculate composite out gaps by taking average of statistical and structural methods separately and compare it with benchmark output gap by taking average of all six methods to identify demand/supply pressure in Pakistan from 1950-2007. The previous studies are unable to compare each method with the economic history. They also remained unable to conduct further analysis such as forecasting of inflation, by using the output gaps, that has been the important objective of central banks in terms of price stability. They did not use quarterly data. Within this context, this study makes the following contributions.

- The study counsels the output gaps using quarterly data by applying the famous statistical and structural techniques and then results of each method are compared with the historical events that have taken place.
- The study forecasts inflation by using different measures of output gaps and then results are compared on the basis of forecast accuracy.

The study consists of five sections. The second section deals with the data. In addition to that, it also describes the econometric techniques used for the estimation of the models. The third section discusses the estimated results based on different statistical and structural approaches. Finally, the last chapter concludes the study.

2. Data and Econometric Methodology

In this study we have used time series data for Pakistan over the period 1960 to 2014. For the analysis quarterly time series data have been used. Data on Real GDP have been taken from various issues of Economic Survey of Pakistan, published by Ministry of Finance, Government of Pakistan and Annual Reports of State Bank of Pakistan. Data on Consumer Price Index have been taken from International Financial Statistic (IFS) of International Monetary Fund (IMF).

Following variables are used in our analysis:

Real GDP is measured as the market value of all final goods and services newly produced domestically. In order to analyze the different frequency of time series data for empirical analysis, quarterly data set from 1972: I to 2003: IV on GDP at constant Market prices at the base of 1980-81 is taken from (Kemal & Arby, 2004). Then, the gap before 1972:1 and after 2003:4 is fulfilled by taking ten year moving average of quarterly weights, which then are multiplied by the annual GDP at constant Market prices at the base of 1999-00 to get quarterly figures. However, Quarterly data set requires the seasonality adjustment that is obtained by using five quarters central moving average method.

Simple moving average formula is given as follow:

$$Y(t) = \frac{(Y(t-1) + Y(t-2) + \dots + Y(t-k))}{k} \quad (2.1)$$

Where, $k=4$

CPI is the most important inflation indicator in Pakistan, which reflects the movement in prices of the urban workers (Feridun, 2006). We use here CPI² that includes prices of 12 major cities. We calculated annual inflation rate from quarterly CPI as follows:-

a1: Inflation is calculated by taking annualized percentage change in CPI as follows:

$$\pi_t = \left(\frac{P_t}{P_{t-h}} - 1 \right) \times 100 \quad (2.2)$$

Where $h = 4$, P and π stands for the price level (CPI) and inflation rate, respectively.

We discuss the methodology of different statistical and structural methods to calculate the output gap.

2.1 Statistical Methods

2.1.1 Linear Time Trend Method (LTT)

LTT method is simple and straightforward. The potential output is calculated from a linear equation of output or log of output on a time trend. The fitted line is taken as potential output while the difference between the potential output and actual output is considered to be the estimate of the output gap. The fitting of linear time trend on output is calculated as follows:

$$Y_t^* = \alpha_0 + \alpha_1 T \quad (2.3)$$

$t = 1, 2, \dots$

Where Y^* and T stand for potential output in natural log form and time trend, respectively. While α_0 and α_1 are estimated coefficients of fitting time trend.

We can use potential output estimate from equation (2.3) to calculate output gap as follows

$$Y_t^g = (Y_t - Y_t^*) * 100 \quad (2.4)$$

$t = 1, 2, \dots$

Where, Y^g and Y stand for the output gap and actual output, respectively.

2.1.2 Quadratic Time Trend Method (QTT)

The output gap by quadratic time trend method is estimated as percentage deviation of real GDP from its quadratic trend. The fitting of Quadratic time trend on GDP is calculated as follows.

$$Y_t^* = \alpha_0 + \alpha_1 T + \alpha_2 T^2 \quad (2.5)$$

$t = 1, 2, \dots$

Where Y^* , T and T^2 stand for potential output, time trend and square of time trend, respectively. While α_0 , α_1 , and α_2 , are estimated coefficients of fitting trend. We can use potential output estimate from equation (2.5) to calculate output gap as follows

$$Y_t^g = \left(\frac{(Y - Y_t^*)}{Y_t^*} \right) * 100 \quad (2.6)$$

² There are also other indicator of inflation's measure such as WPI, SPI and GDP deflator but the mostly usable is CPI (Feridun, 2006).

$t = 1, 2, \dots$

Where Y^g and Y stand for the output gap and actual output, respectively.

2.1.3 Hodrick-Prescott Filter Method (HP Filter)

The HP method (1997) is commonly used to estimate potential output from actual output by fitting a smooth curve along a point. The conceptual framework given by Hodrick-Prescott (1997) is that Let Y_t denote an observable time series. The HP filter decomposes Y into a non-stationary trend " Y^* ", and a stationary residual component, Y^g that is:-

$$Y_t = Y_t^g + Y_t^* \quad (2.7)$$

For $t = 1, 2, \dots, T$.

Where, Y^g and Y^* are unobservable components and Y^* (potential output) is obtained by HP filter that allocate the weight to Y^* against the signal Y . HP solves the problem by minimizing:-

$$\underset{\{Y_t^*\}_{t=1}^T}{Min} \left\{ \sum_{t=1}^T Y_t^{g^2} + \lambda \sum_{t=1}^T [(Y_t^* - Y_{t-1}^*) - (Y_{t-1}^* - Y_{t-2}^*)]^2 \right\} \quad (2.8)$$

Where $Y_t^g = Y_t - Y_t^*$ Let λ be a positive (weight) number, which penalizes the variability in the Y^* series. The larger value of λ will make the solution series smoother. If there is no disturbance then the signal is fully informative and it is better to set $\lambda=0$. If λ approaches infinity, then the function in equation (2.8) is minimized by penalizing changes in potential growth, which is done by making potential output growth constant or Y^* approaches the ordinary least squares estimate of Y against a linear time trend. Hodrick and Prescott (1980) did use $\lambda = 1600$ for quarterly data. We set $\lambda = 1600$ as used by (Hodrick & Prescott, 1997).

2.1.4 The Band Pass Baxter and King Filter (BP)

We use the BP filter developed by (Baxter & King, 1995). The most important contribution of Baxter & King (1995) is the derivation of a band pass filter to estimate directly the cyclical component C_t . The cyclical component C_t is extracted by applying to Y_t as:

$$C_t = \sum_{j=-k}^k \alpha_j Y_{t-j} \quad (2.9)$$

Where, α_j are corresponding weights of the frequency response function. These weights are derived from the inverse Fourier transformation. When using the Baxter & King filter, k observations are lost at the beginning and at the end of the sample period, according to the required degree of approximation to the ideal filter. The common choice is $k = 12$ for quarterly data. In order to reduce the loss of data at the beginning and at the end of the sample, truncated versions of the filter can be used. Alternatively, it is possible to forecast and backcast the series before applying the filter so as to use the complete moving average³.

³ In our analysis, the Band Pass Baxter and King filter is extended by means of a simple, mechanical projection.

2.1.5 Double Exponential Smoothing Method (DES)

The Exponential smoothing technique can be useful in time series data, either to smooth data or to make forecasts which assign exponentially decreasing weights over time. The data series to whom exponential smoothing is applied, represented by $\{Y_t\}$, and the output of the exponential smoothing algorithm is commonly written as $\{S_t\}$. The simplest form of exponential smoothing at time $t=0$ is given by the formulas as follows:

$$S_1 = Y_0$$

$$S_t = \alpha Y_{t-1} + (1 - \alpha)S_{t-1} \quad (2.10)$$

Where α and t stand for the smoothing factor and time period, while $0 < \alpha < 1$ and $t > 1$. The Values of α close to one have less of a smoothing effect and give greater weight to recent changes in the data, while values of α closer to zero have a greater smoothing effect and less responsive to recent changes. There is no formally correct procedure for choosing α . Sometimes, the statistician's judgment chooses an appropriate factor and default value of $\alpha = 0.3$ as a smoothing factor. The smoothed statistic S_t is a simple weighted average of the previous observation Y_{t-1} and the previous smoothed statistic S_{t-1} .

If the trend as well as the mean is varying slowly over time, a higher-order smoothing model is needed to track the varying trend. Simple exponential smoothing does not follow the important component of time series "Trend" that is handled in double exponential smoothing method. The Brown's linear exponential smoothing (LES) model introduced by (Brown, 1963) is the simplest time-varying trend model, which uses two different smoothed series that are centered at different points in time. The standard form of this model is usually expressed as follows: Let S' denote the singly-smoothed series obtained by applying simple exponential smoothing to series Y that is seasonally adjusted⁴ as in equation (2.10):

$$S'(t) = \alpha Y(t) + (1 - \alpha)S'(t-1) \quad (2.11)$$

Then, let S'' denote the double-smoothed series obtained by applying simple exponential smoothing (using the same α) to series S' :

$$S''(t) = \alpha S'(t) + (1 - \alpha)S''(t-1) \quad (2.12)$$

Finally, the forecast $\hat{Y}(t+1)$ is given by:

$$\hat{Y}(t+1) = a(t) + b(t) \quad (2.13)$$

Where:

$$a(t) = 2S'(t) - S''(t) \dots \text{the estimated level at period } t$$

$$b(t) = (\alpha/(1-\alpha))(S'(t) - S''(t)) \dots \text{the estimated trend at period } t$$

2.2 Structural Methods

Statistical methods are easy to use but have some shortcomings as (Quah, 1992) argues that it is impossible to separate the relative importance of demand and supply shocks in an univariate framework. Conversely, Structural methods rely on a specific economic theory (Chagny & Dopke, 2001). This method includes Structural Vector Autoregressive (SVAR) method.

⁴Our data set is seasonally adjusted that does not require the application of triple Exponential Smoothing Method.

2.2.1 Structural Vector Autoregressive Method (SVAR)

In order to apply Structural Vector Autoregressive (SVAR) method, there is need to check the stationarity of variables and cointegration among them. The cointegration is conducted between real output and prices (CPI).

We use a Structural VAR developed by (Blanchard & Quah, 1989). Suppose the variables discussed above are differenced stationary, not cointegrated and can be represented by the Vector autoregressive (VAR). That's why; we use Structural VAR in order to calculate potential output, originally developed by (Blanchard & Quah, 1989). According to them, real GNP is affected both by demand and supply side disturbances, where demand side disturbances have no long-run effect on real GNP. While, on supply side, productivity shocks are assumed to have long run effect on output. In addition to that, univariate model have not the possibility to decompose a variable in to temporary and permanent components but the application of bivariate VAR has the possibility of decomposing a variable into temporary and permanent components (Enders, 2004). We follow the (Blanchard & Quah, 1989) SVAR method by using variables i.e. real GDP and Inflation.

The demand and supply shocks are not observed, but the problem can be handled by the estimation of VAR. Given that the variables are stationary, there exists a VAR representation given that:

$$\begin{bmatrix} \Delta Y_t \\ Z_t \end{bmatrix} = \begin{bmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L) \end{bmatrix} \begin{bmatrix} \Delta Y_{t-1} \\ Z_{t-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \quad (2.14)$$

OR

$$X_t = A(L)X_{t-1} + e_t \quad (2.14a)$$

Whereas X_t , the column vector of $(\Delta Y_t, Z_t)$, e_t the column vector of (e_{1t}, e_{2t}) and $A(L)$ = the 2×2 matrix with elements equal to the polynomials $A_{ij}(L)$ and the coefficient of $A_{ij}(L)$ are denoted by $a_{ij}(k)$ ⁵.

The VAR residuals are composite of the pure innovation ε_{1t} and ε_{2t} . For example, e_{1t} is the one-step-ahead forecast error of Y_t ; i.e. $e_{1t} = \Delta Y_t - E_{t-1} \Delta Y_t$. From the bivariate moving average (BMA), the one-step-ahead forecast error is $b_{11}(0)\varepsilon_{1t} + b_{12}(0)\varepsilon_{2t}$, so we can write it as follows:

$$e_{1t} = b_{11}(0)\varepsilon_{1t} + b_{12}(0)\varepsilon_{2t} \quad (2.15)$$

Similarly, since e_{2t} is the one-step-ahead forecast error of Z_t

$$e_{2t} = b_{21}(0)\varepsilon_{1t} + b_{22}(0)\varepsilon_{2t} \quad (2.16)$$

or, combining (2.15) and (2.16), we get

$$\begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = \begin{bmatrix} b_{11}(0) & b_{12}(0) \\ b_{21}(0) & b_{22}(0) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

If $b_{11}(0), b_{12}(0), b_{21}(0)$ and $b_{22}(0)$ were known, it would be possible to recover ε_{1t} and ε_{2t} from the regression residual e_{1t} and e_{2t} . We need four restrictions to identify these structural shocks. Blanchard and Quah identify these restrictions as follows:

Given (2.15) and noting that $E\varepsilon_{1t}\varepsilon_{2t} = 0$, the normalization $\text{var}(\varepsilon_{1t}) = \text{var}(\varepsilon_{2t}) = 1$ means that the variance of e_{1t} is

$$\text{var}(e_{1t}) = b_{11}(0)^2 + b_{12}(0)^2 \quad (2.17)$$

⁵ For example $A_{11}(L) = a_{11}(0) + a_{11}(1)L + a_{11}(2)L^2 + \dots$

Similarly, using (2.16) the variance of e_{2t} is

$$\text{var}(e_{2t}) = b_{21}(0)^2 + b_{22}(0)^2 \quad (2.18)$$

The covariance of VAR residual is

$$\text{COV}(e_{1t}, e_{2t}) = b_{11}(0)b_{21}(0) + b_{12}(0)b_{22}(0) \quad (2.19)$$

For all possible realization of the $\{\varepsilon_{1t}\}$ sequence, ε_{1t} shocks will have only temporary effects on the ΔY_t sequence if

$$b_{21}(0)[1 - \sum_{a22}(k)] + b_{21}(0)\sum_{a12}(k) = 0 \quad (2.20)$$

If we apply these four restrictions then we can identify structural shocks i.e., ε_{1t} and ε_{2t} . Now the potential output component ΔY^p is determined by cumulating supply-side shocks for example, set all $\{\varepsilon_{1t}\}$ shocks equal to zero and use the actual $\{\varepsilon_{2t}\}$ series to obtain permanent changes in $\{Y_t\}$ as⁶

$$\Delta Y_t^p = \sum_{k=0}^{\infty} b_{12}(k) \varepsilon_{2t-k} \quad (2.21)$$

After calculating potential output we can identify the output gap by taking difference of actual and potential output.

2.3 Inflation Forecasting

We use two empirical approaches to forecast inflation. As a benchmark, we estimate a univariate Autoregressive and Moving Average (ARMA) model. Next, we use a bivariate Vector Autoregressive (VAR) model that includes inflation and output gaps estimated from different statistical and structural filters. Short theoretical descriptions of both models are given below.

2.3.1 Univariate Forecasting Model

In the simplest form, inflation can be modeled as an ARMA process. ARMA models in our study provide a benchmark for inflation's forecasting. By definition, an ARMA model is covariance stationary in that it has a finite and time-invariant mean and covariance. For an ARMA model to be stationary, the characteristic roots of the difference equation must lie inside the unit circle. We will focus on the (Box & Jenkins, 1976) approach to identification, estimation, diagnostic checking, and forecasting a univariate time series.

The equation for an ARMA model is given as:

$$\pi_t = \alpha + \sum_{i=1}^p \phi_i \pi_{t-i} + \sum_{j=0}^q \theta_j \varepsilon_{t-j} \quad (2.22)$$

Where π denote inflation rate and ε denotes white noise error term. Modeling and forecasting of various ARMA time series models based on Pakistan's quarterly Inflation data would be carried out by following methodology based on (Feridun, 2006).

1. Specification and estimation of various possible types of ARMA models through equation (2.22).
2. Obtaining ex-post forecast after empirically estimating the various types of ARMA models.
3. Comparison of forecasting performance of various types of ARMA models by using certain statistical measures.

Firstly, time series will be tested for stationarity both graphically and with formal testing procedures by utilizing Autocorrelation Function (ACF), Partial Autocorrelation

⁶ The lag length criteria and its detail about SVAR method are reported in appendix. A.3

Function (PACF) and Augmented Dickey-Fuller test of unit root (Dickey & Fuller, 1979). If the original series is found to be nonstationary some appropriate transformations will be made for achieving stationarity, otherwise, we will carry on to next step.

Secondly, the next step is to obtain an appropriate model(s) based on (Box & Jenkins, 1976) methodology. Then, the model(s) will be identified by using both ACF and PACF and estimated through OLS method. Moreover, the best fitting model(s) will pass over various residual and normality tests and only qualifying model(s) will be selected and reserved for forecasting purpose. Then we perform out of sample forecast on our selected model(s). This involve making forecast using only data, dated before the forecast period.

Finally, Forecasting performance of the various types of ARMA models would be compared by computing statistics such as Mean Square Error (MSE) or Root Mean Square Error (RMSE). The Root Mean Square Error (RMSE) is defined as:

$$RMSE(n) = \sqrt{(T)^{-1} \sum_{i=1}^T (\pi_i^a - \pi_i^f(n))^2} \quad (2.23)$$

Where $\pi^f(n)$ shows the models prediction for the rate of inflation n steps into the future and π^a shows the corresponding actual inflation. T is the number of forecasts computed.

2.3.2 Vector Autoregressive (VAR) Models

In data description and forecasting, VAR is proven to be powerful and reliable tool (Sims, 1980). In order to conduct the out of sample inflation forecasting, the simple VAR model is used, in which inflation depends on its past values and past values of output gap. The output gap also depends on its past values and past values of inflation. The mathematical form of a VAR is

$$Y_t = A_1 Y_{t-1} + \dots + A_p Y_{t-p} + B X_t + \varepsilon_t \quad (2.24)$$

Where Y_t , X_t and $A_1 \dots A_p$, B stands for k vector of endogenous variables, vector of exogenous variables, and matrices of coefficients to be estimated, respectively. However ε_t , a vector of innovations that may be contemporaneously correlated with each other but are uncorrelated with their own lagged values and all of the right-hand side variables. The procedure of out of sample h steps ahead forecast is computed by estimating the VAR through a given quarter, making the forecast h steps ahead, reestimating the VAR through the next quarter, making the next forecast and so on through the forecast period (Stock & Watson, 2001).

Finally, Forecasting performance of the VAR Model would be compared by computing statistics such as RMSE as given in equation (2.23).

3. Empirical Findings and Interpretation

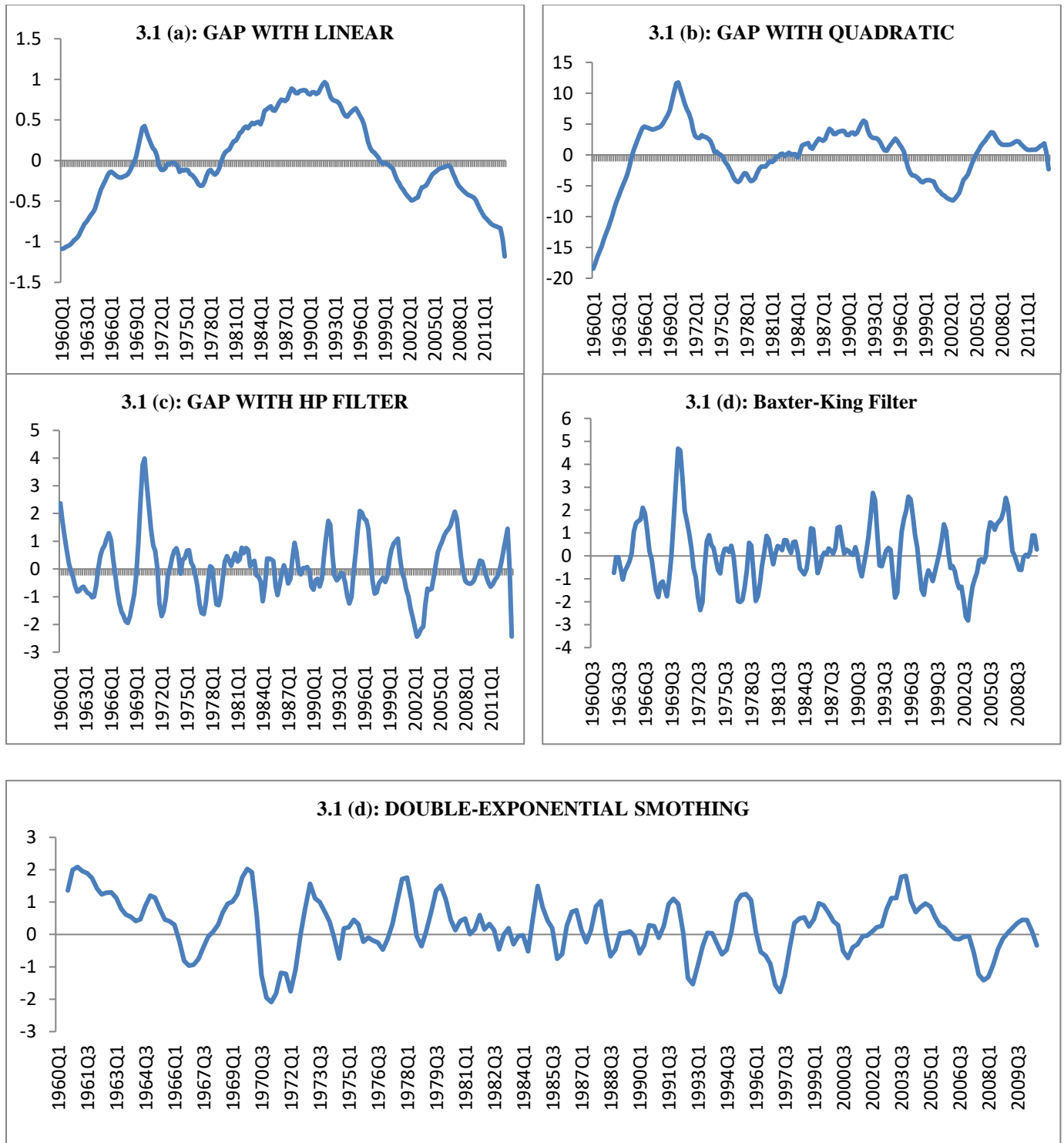
This section contains the results of the estimation along with their interpretation. It is divided in to three sections. In first and second section, we study the results and discussion of output gap. In third section, we describe the results and discussion of inflation forecasting.

3.1 Statistical Methods

These methods of estimation are based on particular statistical procedures and not on specific economic theory. The advantage of these methods is that they do not require much information, and they can be applied even when only one data series is known (Menashe & Yakhin, 2004).

The results of five statistical methods are shown in Figure 3.1

Fig 3.1: Quarterly Output Gap Calculated from Statistical Methods



Discussions

The gap from linear and quadratic trend as show in fig 3.1 (a, b) are almost similar in fluctuations but quadratic is more smoother then the linear filter. The linear and quadratic method indicates that at the beginning of sixteen, Pakistan economy is in recession as from 1960: III to 1970: III. However, at quarter four of 1960, there is strong evidence of trough in

economic activity as output gap is -12.73% and -17.52% calculated from linear and quadratic trend, respectively but HP and exponential smoothing shows that economy is in boom as the output gap is 1.72% and 1.36%, respectively. All mentioned methods show that at 1970: III economy reaches its peak as the output gap calculated from linear, quadratic, HP, BP and exponential smoothing method are 5.23%, 11.75%, 3.98%, 4.60% and 0.58%, respectively, strongly support to the boom period. After that all method shows almost consistent results i.e. economy is in recessionary phase till 1977. At the same period, economy reaches its trough as the output gap in 1977: III from linear, quadratic, HP and BP are -3.88%, -4.43%, -1.63% and -1.91, respectively. While the exponential smoothing method shows that economy reaches its minimum point in 1971: II as the gap is -2.09. However, output gap is also negative in 1977. After 1977 trough economy starts recovering. The linear, quadratic, HP and BP filter identify that it takes almost 15 years to recover. These method shows that economy is in expansion from 1978: III to 1992: II. It reaches peak point in 1992: II as gap in this period from linear, quadratic; HP, BP and exponential smoothing method are 12.91%, 5.57%, 1.73%, 2.75% and 0.94%, respectively. However after 1992 to 2002: IV, economy is in recession. In 2002: IV economy again reaches its trough point as the output gap in linear, quadratic, HP and BP are -6.84%, -7.42%, -2.44% and -2.66%, respectively. After 2002, all methods show that economy is recovering till 2007. All methods, except for the linear detrending, identify that economy is in expansionary phase from 2002 to 2007. However, in 2008 to present economy is in recessionary face that is consistent with historical analysis.

Pakistan was considered a model for economic development during the 1960s in the world, as its GDP approached Rs. 447876.3 million. In the sixties, government priorities shifted from the industrial sector towards the agricultural sector resulting in rapid growths for each along with significant increase in per capita income. After the 1965 war, there was a decrease in the inflow of foreign loans, slow down in the industrial growth, and increase in defense expenditures. The GDP growth was 6.8% on average. The huge trade deficit was financed mainly through foreign loans.

After the separation of eastern Pakistan in 1971, deterioration of economic growth continued to prevail and the country was faced with the challenges of rehabilitation of a post-war economy, high rates of inflation and shrinking agriculture and industrial sectors. In the seventies, economic management shifted towards nationalization. In the early 1972, the government took some drastic steps and brought fundamental structural reforms. Although these reforms were introduced to improve the efficiency of the manufacturing sector, the share of both the manufacturing and agriculture to GDP declined. Due to a decline in the two major sectors, the entire economy saw a decline in GDP growth to 3.6 percent per annum.

In 1990s, dwindle in economic performance of Pakistan appeared as GDP growth rate was 4.4% on average throughout this period. Both the fiscal and current account deficits caused uncontrollable levels of public debt. Fiscal deficit as a ratio of GDP remained above 6%, while the current account deficit in 1990s was 5.9 percent of GDP as compared to 2.7 percent of GDP in 1980s. Pakistan, as a consequence of the nuclear tests conducted in May 1998 by both India and Pakistan, had to face financial hardships brought on by the imposition of international economic sanctions. As a result of the sanctions, capital inflows ceased entirely which caused the economic conditions to become severely difficult.

On average, yearly growth through the period of 2000-2007 remained 7.0%. In 2000-07 real GDP of Pakistan increased from \$60 billion to \$170 billion, with per capita income rising from under \$500 to over \$1000. The volume of international trade increased from \$20 billion to \$60 billion.

In the 2000s, more liberal economic strategies have been adopted with the aim to enhance the share of Pakistan's exports in the world economy. The privatization process started with a focus on banking, telecommunication, oil and gas and energy sectors. More

importantly, in the aftermath of 9/11, the foreign grants of about \$1 billion to \$1.5 billion per annum during 2003-08 come in after easing of sanctions. Foreign exchange reserves increased due to remittances amounting to \$18.5 billion during 2003-08. The current account surplus was recorded at an annual average of 1.9% of the GDP. The rising remittances since September 11, 2001 onward kept the current account balance in surplus.

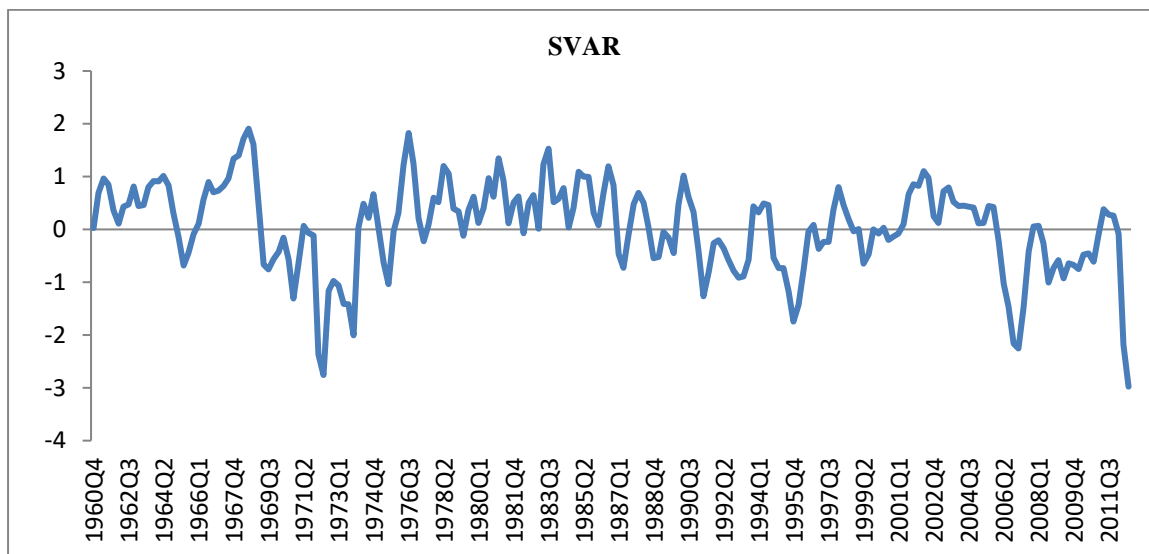
The floods of 2010 and 2011, decline in security situation, uncertainty and lack of focus are one of the reasons behind Pakistan poor economic performance in last five years. After 2007 international prices of food and fuel prices start increasing, and energy crisis of Pakistan's initiate began to visible itself, the momentum of growth was break up. The average GDP growth rate (annually) 3 percent over the period being discussed is less as compared to the period from FY 2003 to FY 2007 in which it was 7%.

In sum up, we can say that the economy of Pakistan since 1960 has pass through three complete business cycles that are strongly supported by the historical view point discussed above (Mahmood, 2008). Furthermore, again we have noted that quadratic time trend method produce the results which are consistent with the history. The second best method, here, is found linear time trend method. However this method produced counterfactual results at the end period of sample.

3.2 Structural Methods

These method assume that particular theory exists which explains economic behavior properly (Menashe & Yakhin, 2004).

Fig 3.2: Quarterly Output Gap Calculated from Structural Methods



Discussions

In first half of 1960s SVAR method indicates increasing growth trend in economy. In second half, economy is expanding and reached its peak point in 1970: I as the gap calculated from is 1.91%. After 1970, the results show recessionary phase. Moreover, results show that in the decade of 1980s economy is in expansionary phase. It fell again in to recession in 1990s and it is in expansionary phase from 2001: I to 2007: II. However, in 2007: III to present economy is in recessionary face.

In sum up, we can say that SVAR method explains that the economy of Pakistan, since 1960, has passed through three complete business cycles that are strongly supported by the historical view point.

3.3 Results and Discussion of Inflation Forecasting

This section discusses the comparison of results based on forecast comparison between different ARMA and VAR models. The results of quarterly inflation forecast comparison between models using inflation as annualized percentage change in CPI are given in table 3.1

Table 3.1 Forecast Comparison in Quarterly Data

Quarterly out of Sample 1-Step Ahead Forecast from 2012: II to 2014: I			
S.NO	Methods	MSE	RMSE
ARMA			
1	ARMA(8,4)	1.42	1.19
Statistical			
1	Linear Time Trend (LTT)	4.14	2.04
2	Quadratic Time Trend (QTT)	4.30	2.07
3	Hodrick-Prescott (HP filter)	4.47	2.11
4	Band Pass Baxter and King Method Filter (BP)	5.19	2.28
5	Double Exponential Smoothing (DES)	4.47	2.11
Structural			
1	Structural Vector Autoregressive (SVAR)	3.39	1.84

Discussions

The results indicate that the forecast ability of the output gap model based on the structural method (i.e. SVAR) gives better results than the statistical methods as the mean square error (MSE) and root mean square error (RMSE), are minimum in structural method as compare to statistical methods. Moreover, forecast ability of the output gap model based on Structural Vector Autoregressive (SVAR) gives better results both in all statistical and structural methods, as shown in above table. It is consistent with the empirical studies [For Example (Chagny & Dopke, 2001), (Bjørnland, Brubakk, & Jore, 2005) and (Cesaroni, 2010)].

However, the univariate ARMA model again perform better forecast than bivariate VAR models do as the value of MSE and RMSE is minimum in case of ARMA (8,4) model that is consistent with the empirical studies.

4. Summary and Conclusion

In order to calculate output gap, five statistical and one structural methods are used, where statistical methods consist of Linear Time Trend, Quadratic Time Trend, Hodrick-Prescott (HP filter), Band Pass Baxter and King Filter (BP) and Exponential Smoothing method and the structural methods include Structural Vector Autoregressive (SVAR) method. The analysis is conducted on the basis of quarterly data set. The results show that the economy of Pakistan since 1960 has gone through three complete business cycles, which is strongly supported by the historical facts and empirical evidence (see (Mahmood, 2008).

According to the comparison of statistical measures of output gap show that quadratic detrending method produces the results which are consistent with the historical facts. The

second best method, here, is found linear detrending method. However, this method produced counterfactual results at the end period of sample period.

According to the comparison of structural measures of output gap (i.e. SVAR) is also consistent with the economic history of Pakistan. The comparisons of output gap to forecast inflation indicate that structural methods perform better than the statistical methods. Moreover, the Structural Vector Autoregressive (SVAR) is the best model to forecast inflation as compare to all others methods. The Linear Time Trend (LTT) is the second best model which forecast inflation better than the other methods do. The results indicate that univariate ARMA model forecasts inflation better than that of the bivariate VAR models using any of the output gap measure.

References

- Arby, M. F. (2001). Long-run trend, business cycles and short-run shocks in real GDP. *SBP Working Paper Series No. 01*.
- Baxter, M., & King, R. (1995). Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series. *NBER Working Papers, Cambridge*.
- Bjørnland, H. C., Brubakk, L., & Jore, A. S. (2005). The Output Gap in Norway A Comparison of Different Methods. *Norges Bank Economic Bulletin, 76(2)*, 90-100.
- Blanchard, O. J., & Quah, D. (1989). The dynamic effect of aggregate demand and supply disturbances. *American Economic Review, 79(4)*, 655-673.
- Box, G., & Jenkins, G. (1976). Time Series Analysis: Forecasting and Control.
- Brown, R. G. (1963). Smoothing, Forecasting and Prediction of Discrete Time Series.
- Cesaroni, T. (2010). Estimating potential output using business survey data in a SVAR framework. *Economics Bulletin, 30(3)*, 2249-2258.
- Chagny, O., & Dopke, j. (2001). Measures of the Output Gap in the Euro-Zone: An Empirical Assessment of Selected Methods. *Kiel Institute of World Economics. Kiel Working Paper, 1053*.
- De Brouwer, G. (1998). "Estimating Output Gaps. *Research Discussion Paper No. 9809 Australia: Reserve Bank of Australia*.
- Dickey, D., & Fuller, A. W. (1979). Distribution of the estimators for autoregressive time series with a unit-root. *Journal of the American Statistical Association, 74*, 427-431.
- Enders, W. (2004). *Applied Econometric Time Series*. John Wiley & Sons, Inc.
- Feridun. (2006). Forecasting Inflation through Econometric Models: An Empirical Study on Pakistani Data. *Mete Dogus University Journal, 7(1)*, 39-47.
- Haider, A., & Safdar Ullah, K. (2008). Estimating Output Gap for Pakistan Economy: Structural and Statistical Approaches. *MPRA Paper 9736, University Library of Munich, Germany*.
- *Handbook of Statistics on Pakistan's Economy*. (2010).
- Hodrick, R., & Prescott, E. (1997). Post-War US Business Cycles: An Empirical Investigation. *Journal of Money, Credit and Banking, 29(1)*, 1-16.
- Kemal, A. R., & Arby, F. (2004). Quarterisation of Annual GDP of Pakistan. *Pakistan Institute of Development Economics*.
- Mahmood, e. a. (2008). Evaluation of Macro Economic Policies of Pakistan 1950-2008. *Journal of Political Studies, 14*, 58-75.
- Menashe, Y., & Yakhin, Y. (2004). Mind the Gap: Structural and Nonstructural Approaches to Estimating Israel's Output Gap. *Israel Economic Review, 2(2)*.
- Nelson, C., & Plosser, c. (1982). Trends and Random Walks in Macroeconomic series. *Journal of Monetary Economics, 10*, 139-162.

- Quah, D. (1992). The Relative Importance of Permanent and Transitory Components: Identification and some Theoretical Bounds. *Econometrica*, 60, 107-118.
- Sims, C. A. (1980). Macroeconomics and Reality. *Econometrica*, 48(1), 1-48.
- Stock, J. H., & Watson, M. W. (2001). Vector Autoregressions. *Journal of Economic Perspectives*, 15(4), 101-115.

Table A.8.2, Results of quarterly output gap by statistical and structural methods

YEARS	LTTG	QTTG	HPG	BPG	DESG	SVARG
1960Q4	-12.73	-17.52	1.72	-	1.36	-
1961Q1	-12.56	-16.55	1.21	-	1.99	-
1961Q2	-12.43	-15.64	0.69	-	2.09	-
1961Q3	-12.29	-14.75	0.21	-	1.95	-
1961Q4	-11.98	-13.74	-0.07	-	1.89	-
1962Q1	-11.63	-12.73	-0.29	-	1.74	-
1962Q2	-11.39	-11.83	-0.60	-	1.42	0.03
1962Q3	-11.07	-10.89	-0.82	-	1.23	0.69
1962Q4	-10.50	-9.77	-0.79	-	1.29	0.97
1963Q1	-9.87	-8.60	-0.68	-0.74	1.30	0.85
1963Q2	-9.31	-7.53	-0.64	-0.05	1.13	0.36
1963Q3	-8.93	-6.64	-0.76	-0.07	0.80	0.11
1963Q4	-8.52	-5.76	-0.86	-0.54	0.62	0.43
1964Q1	-8.07	-4.86	-0.91	-1.03	0.55	0.47
1964Q2	-7.68	-4.06	-1.02	-0.66	0.42	0.81
1964Q3	-7.17	-3.16	-1.00	-0.46	0.47	0.44
1964Q4	-6.27	-1.89	-0.58	-0.23	0.88	0.46
1965Q1	-5.19	-0.49	0.02	0.12	1.20	0.80
1965Q2	-4.26	0.76	0.49	1.07	1.14	0.91
1965Q3	-3.59	1.74	0.72	1.44	0.78	0.91
1965Q4	-3.03	2.57	0.84	1.53	0.46	1.01
1966Q1	-2.36	3.49	1.10	1.59	0.41	0.83
1966Q2	-1.78	4.30	1.29	2.10	0.30	0.31
1966Q3	-1.65	4.62	1.03	1.84	-0.21	-0.14
1966Q4	-1.93	4.47	0.38	1.10	-0.81	-0.68
1967Q1	-2.16	4.35	-0.20	0.21	-0.97	-0.45
1967Q2	-2.38	4.21	-0.77	-0.14	-0.94	-0.09
1967Q3	-2.54	4.12	-1.25	-0.86	-0.75	0.11
1967Q4	-2.50	4.21	-1.52	-1.52	-0.39	0.56
1968Q1	-2.37	4.37	-1.70	-1.80	-0.07	0.90
1968Q2	-2.26	4.49	-1.89	-1.19	0.09	0.70
1968Q3	-2.03	4.71	-1.94	-1.12	0.30	0.73
1968Q4	-1.51	5.22	-1.70	-1.46	0.68	0.83
1969Q1	-0.87	5.83	-1.32	-1.77	0.95	0.96
1969Q2	-0.23	6.44	-0.92	-1.01	1.02	1.34
1969Q3	0.66	7.30	-0.23	0.00	1.24	1.40
1969Q4	2.04	8.68	0.98	1.51	1.76	1.72
1970Q1	3.55	10.19	2.38	3.14	2.02	1.91
1970Q2	4.96	11.61	3.75	4.68	1.92	1.61
1970Q3	5.23	11.75	3.98	4.60	0.58	0.48
1970Q4	4.30	10.55	3.03	3.38	-1.26	-0.67
1971Q1	3.43	9.41	2.19	1.96	-1.96	-0.75
1971Q2	2.59	8.30	1.43	1.52	-2.09	-0.57
1971Q3	1.89	7.34	0.85	0.99	-1.83	-0.42
1971Q4	1.54	6.76	0.65	0.35	-1.18	-0.16
1972Q1	0.73	5.68	0.01	-0.54	-1.21	-0.57
1972Q2	-0.74	3.92	-1.25	-0.94	-1.76	-1.31

1972Q3	-1.41	3.00	-1.70	-1.82	-1.08	-0.64
1972Q4	-1.44	2.74	-1.52	-2.37	-0.02	0.07
1973Q1	-1.19	2.76	-1.06	-2.02	0.80	-0.06
1973Q2	-0.52	3.21	-0.17	-0.44	1.57	-0.12
1973Q3	-0.54	2.94	0.03	0.65	1.12	-2.36
1973Q4	-0.36	2.87	0.42	0.91	1.00	-2.76
1974Q1	-0.35	2.63	0.65	0.48	0.71	-1.16
1974Q2	-0.45	2.26	0.75	0.35	0.40	-0.98
1974Q3	-0.90	1.56	0.50	-0.14	-0.11	-1.06
1974Q4	-1.75	0.44	-0.17	-0.61	-0.74	-1.41
1975Q1	-1.43	0.50	0.33	-0.77	0.19	-1.42
1975Q2	-1.53	0.14	0.39	-0.03	0.22	-2.01
1975Q3	-1.41	0.01	0.66	0.30	0.45	0.03
1975Q4	-1.51	-0.35	0.68	0.31	0.31	0.49
1976Q1	-2.06	-1.16	0.22	0.18	-0.22	0.22
1976Q2	-2.25	-1.61	0.10	0.44	-0.09	0.67
1976Q3	-2.62	-2.22	-0.22	0.01	-0.18	0.02
1976Q4	-3.04	-2.88	-0.62	-0.95	-0.25	-0.64
1977Q1	-3.67	-3.73	-1.27	-1.97	-0.47	-1.04
1977Q2	-3.92	-4.23	-1.58	-2.01	-0.15	-0.04
1977Q3	-3.88	-4.43	-1.63	-1.91	0.33	0.31
1977Q4	-3.35	-4.16	-1.24	-1.38	1.03	1.22
1978Q1	-2.41	-3.49	-0.48	-0.62	1.71	1.83
1978Q2	-1.62	-2.97	0.10	0.58	1.75	1.25
1978Q3	-1.45	-3.04	0.04	0.46	1.02	0.21
1978Q4	-1.92	-3.73	-0.70	-0.75	-0.05	-0.22
1979Q1	-2.21	-4.23	-1.27	-1.96	-0.36	0.09
1979Q2	-1.93	-4.18	-1.31	-1.75	0.10	0.60
1979Q3	-1.29	-3.79	-1.02	-1.13	0.69	0.52
1979Q4	-0.24	-3.00	-0.34	-0.39	1.36	1.20
1980Q1	0.73	-2.27	0.26	0.19	1.51	1.05
1980Q2	1.32	-1.90	0.46	0.87	1.09	0.39
1980Q3	1.55	-1.89	0.30	0.72	0.46	0.34
1980Q4	1.77	-1.88	0.12	0.12	0.13	-0.12
1981Q1	2.42	-1.44	0.37	-0.36	0.41	0.37
1981Q2	3.02	-1.05	0.57	0.11	0.49	0.62
1981Q3	3.11	-1.15	0.27	0.43	0.01	0.12
1981Q4	3.57	-0.89	0.34	0.39	0.16	0.40
1982Q1	4.37	-0.27	0.77	0.25	0.59	0.97
1982Q2	4.55	-0.27	0.57	0.69	0.16	0.62
1982Q3	5.10	0.10	0.76	0.68	0.33	1.35
1982Q4	5.38	0.21	0.69	0.34	0.14	0.93
1983Q1	5.14	-0.19	0.10	0.13	-0.46	0.11
1983Q2	5.60	0.10	0.22	0.59	0.00	0.51
1983Q3	6.00	0.35	0.29	0.62	0.19	0.63
1983Q4	5.80	0.00	-0.23	0.17	-0.30	-0.07
1984Q1	6.07	0.12	-0.28	-0.54	-0.04	0.50
1984Q2	6.19	0.11	-0.46	-0.67	-0.03	0.65
1984Q3	5.80	-0.42	-1.16	-0.80	-0.52	0.01
1984Q4	6.63	0.29	-0.63	-0.58	0.53	1.23
1985Q1	7.94	1.48	0.37	0.09	1.50	1.53
1985Q2	8.23	1.66	0.38	1.21	0.84	0.52
1985Q3	8.47	1.79	0.34	1.17	0.43	0.57
1985Q4	8.70	1.92	0.29	0.16	0.20	0.78
1986Q1	8.09	1.19	-0.60	-0.75	-0.75	0.03
1986Q2	8.00	1.01	-0.94	-0.47	-0.60	0.41
1986Q3	8.65	1.58	-0.56	-0.04	0.27	1.09
1986Q4	9.30	2.16	-0.16	0.14	0.70	1.00
1987Q1	9.83	2.63	0.14	0.06	0.75	1.00
1987Q2	9.77	2.50	-0.15	0.32	0.14	0.31

1987Q3	9.62	2.27	-0.52	0.23	-0.24	0.08
1987Q4	9.99	2.59	-0.36	0.10	0.13	0.70
1988Q1	10.89	3.47	0.35	0.36	0.86	1.19
1988Q2	11.66	4.22	0.95	1.23	1.03	0.84
1988Q3	11.47	3.98	0.59	1.27	0.09	-0.46
1988Q4	10.96	3.42	-0.07	0.63	-0.68	-0.73
1989Q1	10.96	3.38	-0.19	0.10	-0.47	-0.10
1989Q2	11.30	3.71	0.03	0.27	0.03	0.48
1989Q3	11.39	3.79	0.03	0.22	0.05	0.69
1989Q4	11.51	3.90	0.07	0.08	0.10	0.50
1990Q1	11.45	3.83	-0.06	0.05	-0.06	0.05
1990Q2	10.91	3.27	-0.63	0.38	-0.58	-0.55
1990Q3	10.82	3.18	-0.75	0.07	-0.33	-0.52
1990Q4	11.18	3.56	-0.40	-0.57	0.28	-0.05
1991Q1	11.21	3.62	-0.36	-0.90	0.25	-0.16
1991Q2	10.91	3.33	-0.63	-0.37	-0.10	-0.45
1991Q3	11.12	3.58	-0.37	0.18	0.26	0.47
1991Q4	11.85	4.37	0.43	0.83	0.94	1.02
1992Q1	12.46	5.05	1.15	1.74	1.10	0.62
1992Q2	12.91	5.57	1.73	2.75	0.94	0.33
1992Q3	12.62	5.32	1.59	2.42	0.05	-0.43
1992Q4	11.33	4.03	0.47	0.96	-1.35	-1.27
1993Q1	10.42	3.15	-0.25	-0.41	-1.54	-0.82
1993Q2	10.03	2.82	-0.40	-0.45	-0.96	-0.26
1993Q3	9.86	2.72	-0.34	-0.10	-0.38	-0.20
1993Q4	9.78	2.72	-0.15	0.23	0.05	-0.36
1994Q1	9.48	2.49	-0.17	0.35	0.03	-0.57
1994Q2	8.85	1.94	-0.49	0.26	-0.29	-0.78
1994Q3	8.02	1.20	-0.99	-0.81	-0.61	-0.91
1994Q4	7.43	0.70	-1.24	-1.81	-0.48	-0.89
1995Q1	7.30	0.68	-1.01	-1.60	0.10	-0.58
1995Q2	7.78	1.28	-0.16	-0.07	1.00	0.44
1995Q3	8.12	1.74	0.59	1.08	1.22	0.32
1995Q4	8.49	2.23	1.38	1.61	1.25	0.49
1996Q1	8.74	2.63	2.09	1.98	1.06	0.46
1996Q2	8.22	2.22	2.03	2.58	0.10	-0.54
1996Q3	7.51	1.64	1.81	2.46	-0.53	-0.73
1996Q4	6.94	1.20	1.76	1.77	-0.65	-0.73
1997Q1	6.11	0.51	1.45	0.92	-0.90	-1.17
1997Q2	4.65	-0.79	0.53	0.37	-1.56	-1.75
1997Q3	3.15	-2.11	-0.40	-0.60	-1.78	-1.43
1997Q4	2.11	-2.97	-0.88	-1.48	-1.29	-0.76
1998Q1	1.59	-3.31	-0.84	-1.70	-0.41	-0.04
1998Q2	1.36	-3.37	-0.52	-0.98	0.36	0.08
1998Q3	0.91	-3.63	-0.42	-0.63	0.49	-0.37
1998Q4	0.48	-3.87	-0.31	-0.83	0.53	-0.24
1999Q1	-0.20	-4.34	-0.47	-1.10	0.25	-0.23
1999Q2	-0.50	-4.45	-0.26	-0.68	0.48	0.37
1999Q3	-0.40	-4.16	0.34	-0.28	0.97	0.80
1999Q4	-0.51	-4.07	0.70	0.15	0.91	0.46
2000Q1	-0.74	-4.10	0.91	0.69	0.68	0.20
2000Q2	-1.08	-4.22	1.00	1.38	0.41	-0.04
2000Q3	-1.39	-4.31	1.10	1.10	0.28	0.01
2000Q4	-2.41	-5.07	0.44	0.16	-0.51	-0.65
2001Q1	-3.32	-5.72	-0.13	-0.52	-0.73	-0.48
2001Q2	-3.85	-6.01	-0.36	-0.44	-0.40	0.00
2001Q3	-4.49	-6.39	-0.74	-0.65	-0.30	-0.08
2001Q4	-4.97	-6.61	-0.99	-1.12	-0.06	0.03
2002Q1	-5.55	-6.93	-1.40	-1.41	-0.04	-0.20
2002Q2	-6.03	-7.14	-1.74	-1.34	0.09	-0.13

2002Q3	-6.43	-7.28	-2.06	-1.92	0.23	-0.08
2002Q4	-6.84	-7.42	-2.44	-2.66	0.26	0.10
2003Q1	-6.73	-7.08	-2.35	-2.82	0.79	0.67
2003Q2	-6.49	-6.60	-2.16	-1.97	1.12	0.86
2003Q3	-6.31	-6.19	-2.08	-1.35	1.13	0.82
2003Q4	-5.40	-5.07	-1.32	-1.01	1.78	1.11
2004Q1	-4.64	-4.08	-0.71	-0.73	1.81	0.98
2004Q2	-4.52	-3.69	-0.76	-0.19	1.03	0.25
2004Q3	-4.29	-3.20	-0.71	-0.15	0.69	0.12
2004Q4	-3.72	-2.36	-0.31	-0.28	0.84	0.72
2005Q1	-3.04	-1.41	0.18	-0.02	0.96	0.80
2005Q2	-2.46	-0.53	0.61	0.97	0.87	0.52
2005Q3	-2.11	0.11	0.82	1.47	0.53	0.44
2005Q4	-1.84	0.70	0.98	1.39	0.28	0.45
2006Q1	-1.52	1.33	1.22	1.14	0.20	0.43
2006Q2	-1.33	1.85	1.36	1.39	0.04	0.41
2006Q3	-1.25	2.25	1.43	1.51	-0.13	0.12
2006Q4	-1.14	2.70	1.58	1.63	-0.15	0.12
2007Q1	-0.97	3.21	1.83	1.96	-0.07	0.44
2007Q2	-0.85	3.67	2.06	2.53	-0.06	0.43
2007Q3	-1.26	3.59	1.81	2.18	-0.55	-0.21
2007Q4	-2.18	2.99	1.07	1.14	-1.24	-1.03
2008Q1	-3.07	2.43	0.42	0.19	-1.42	-1.46
2008Q2	-3.89	1.95	-0.13	0.00	-1.31	-2.16
2008Q3	-4.50	1.68	-0.44	-0.29	-0.93	-2.25
2008Q4	-4.90	1.64	-0.51	-0.60	-0.44	-1.48
2009Q1	-5.27	1.62	-0.53	-0.61	-0.14	-0.42
2009Q2	-5.63	1.62	-0.52	-0.07	0.05	0.06
2009Q3	-5.92	1.69	-0.41	0.05	0.21	0.07
2009Q4	-6.13	1.85	-0.22	-0.02	0.36	-0.26
2010Q1	-6.30	2.06	0.05	0.19	0.45	-1.00
2010Q2	-6.49	2.25	0.30	0.90	0.44	-0.72
2010Q3	-6.98	2.14	0.27	0.89	0.11	-0.58
2010Q4	-7.75	1.74	-0.03	0.28	-0.34	-0.93
2011Q1	-8.47	1.40	-0.28	-0.33	-0.48	-0.64
2011Q2	-9.19	1.06	-0.51	-0.35	-0.49	-0.67
2011Q3	-9.83	0.82	-0.64	-0.62	-0.36	-0.75
2011Q4	-10.25	0.79	-0.55	-1.00	-0.05	-0.48
2012Q1	-10.61	0.83	-0.40	-0.84	0.17	-0.45
2012Q2	-11.02	0.82	-0.29	0.51	0.22	-0.61
2012Q3	-11.39	0.86	-0.13	1.54	0.27	-0.11
2012Q4	-11.59	1.07	0.22	1.78	0.46	0.38
2013Q1	-11.70	1.38	0.65	1.65	0.60	0.28
2013Q2	-11.89	1.61	1.03	2.27	0.54	0.26
2013Q3	-12.03	1.89	1.46	3.54	0.52	-0.08
2013Q4	-14.01	0.31	0.03	5.80	-1.36	-2.18
2014Q1	-17.09	-2.31	-2.43	8.52	-3.21	-2.98

Source: self calculation and estimation

Where LTTG, QTTG, HPG, DESG, BPG, SVARG, PFG are output gap from Linear Time Trend method, output gap from Quadratic Time Trend method, output gap from Hodrick-Prescott (HP filter), output gap from Baxter-King Method (BP Filter), output gap from double Exponential Smoothing method, output gap from Structural Vector Autoregressive method and output gap from Production Function method, respectively.